DHH-003-016402 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May – 2015

Maths : CMT-4002 : Integration Theory

Faculty Code : 003

Subject Code : 016402

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Answer all questions. Each question carries 14 marks.

(ii) The numbers on the right indicates marks alotted to the question.

1 Answer any seven questions. Choose the correct answer. **2x7=14**

(1) if μ is the Lebesgue measure on [0,2] and ν is the atomic measure concentrated at 1 then [0,1] is ______ set w.r.t. $\mu - \nu$.

(A) a positive set

(B) a negative set

(C) neither a positive set nor a negative set

(D) not measurable.

(2) If $X^* = X \cup \{p\}$ is the one point compactification of an uncountable discrete space X then $\{p\}$ is ______.

(A) a baire set (B) a G_δ – set

(C) not a G_δ – set (D) not a compact set

(3) The counting measure on $\mathbb{R}$ is ______.

(A) not complete (B) σ – finite

(C) finite (D) not σ – finite

(4) ______ is a G_δ – set.

(A) every closed set in a metric space

(B) every closed set in a compact Hausdorff space

(C) every compact set in a locally compact Hausdorff space.

(D) every closed set in a locally comapct Hausdorff space.

(5) Every Baire set in $\mathbb{R}$ has finite measure w.r.t. ______.

(A) Lebesgue measure

(B) every Baire measure on the real line

(C) counting measure

(D) every Borel measure on $\mathbb{R}$

(6) The cumulative distribution function F of a finite Baire measure on the real line is ______.

(A) continuous on the left

(B) continuous

(C) continuous on the right

(D) uniformly continuous

(7) For a locally compact Hausdorff space X, Ba(X) is the σ – algebra generated by ______ in X.

(A) compact sets

(B) compact G_δ sets

(C) closed sets

(D) open sets

(8) If (X, A, μ) is a complete measure space then {s/s is simple measurable on X and $\mu \ \{x \in X \mid s(x) \neq 0\} < \infty\}$ is dense in ______.

(A) $L_\infty(X, A, \mu)$

(B) $L_\rho(X, A, \mu)$, $1 < p \leq \infty$

(C) $L_\rho(X, A, \mu)$, $1 \leq p \leq \infty$

(D) $L_\rho(X, A, \mu)$, $1 \leq p < \infty$

(9) ______ is a true statement.

(A) the counting measure on $\mathbb{R}$ is a Baire measure on the real line

(B) Lebesgue decomposition of a measure space w.r.t. a signed measure

(C) Bo(X)=Ba(X), $\forall$ compact Hausdorff spaces X

(D) Bo(X) is the smallest $\sigma-$algebra containing all closed sets in X

(10) ______ is not a true statement.

(A) Jordan decomposition is unique

(B) Bo(X)=Ba(X) $\forall$ compact Hausdorff spaces X

(C) every $\sigma-$compact set in a locally compact Hausdorff space is $\sigma-$bdd

(D) The Lebesgue measure on $\mathbb{R}$ is $\sigma-$finite.

2 Answer any two : **2x7=14**

(a) Define measure on a measurable space. If X is any set then prove that $\mu:\mathbb{P}(x)\to[0,\infty]$ defined by

$$\mu(A)=\begin{cases}\text{the number of elements if } A\in\mathbb{P}(x) \text{ is finite}\\ \infty \; if \;\; A\in\mathbb{P}(x) \; is \text{ infinite}\end{cases}$$

is a measure on $(X,\mathbb{P}(x))$.

(b) Define signed measure on a measurable space. If μ_1,μ_2 are two measures on a measurable space $(X,\mathbb{A})$ then state and prove the condition under which $\mu_1,-\mu_2$ is a signed measure on $(X,\mathbb{A})$.

(c) State and prove Lebesgue decomposition theorem for a $\sigma-$finite measure w.r.t. another $\sigma-$finite measure on a measurable space.

3 (a) State, without proof, Radon Nikodym theorem for measures. Give an example to show that $\sigma-$finite assumption in the theorem can not be droped. **7**

(b) If X is a countable set and μ is the counting measure on $(X,\mathbb{P}(x))$ then prove that $\underline{|p}(X,\mathbb{P}(x),\mu)\cong l^P$, $\forall\, 1\le P\le\infty$. **7**

OR

(c) Define cumulative distribution function of a finite Baire measure on the real line. State and prove two of its properties. **7**

(d) State, without proof, Caratheodary extension theorem. Give an example to show that σ-finite assumption in the theorem can not be droped. **7**

4 Answer any two : **2x7=14**

(a) For any two measures μ, ν on a measure space $(X, \mathbb{A}, \mu)$, explain the notations $\mu << \nu$ and $\mu \perp \nu$. If $\mu << \nu$ and $\mu \perp \nu$ then prove that $\mu = 0$.

(b) Let μ^* be the outer measure on a set X induced by a measure μ on an algebra $\mathbb{A}$ of subsets of X. Then prove that every element of $\mathbb{A}$ is μ^*-measurable.

(c) Define null set w.r.t. a signed measure μ on $(X, \mathbb{A})$. Give an example of $A \in \mathbb{A}$ s.t. $\mu(A) = 0$ but A is not a null set.

5 Answer any two : **2x7=14**

(a) State, without proof, Fubini's theorem. Give on example to show that the integrability of f in the Fubini's theorem can not be droped (with proof).

(b) Let X be a locally compact Hausdorff space. Then prove that Ba(X) = the σ-algebra generated by compact G_δ sets in X.

(c) Define σ-compact set, Baire set in a locally compact Hausdorff space. Prove that every σ-compact open set in X is a Baire set.

(d) Prove that every Baire measure on a locally compact σ-compact Hausdorff space is regular. Give an example to show that σ-compact assumption can not be droped.